

**BEYOND THE PARTON CASCADE MODEL:  
Klaus Kinder-Geiger and VNI**

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I review Klaus Kinder-Geiger's contributions to the physics of relativistic heavy ion collisions, in particular, the Parton Cascade Model. Klaus developed this model in order to provide a QCD-based description of nucleus-nucleus reactions at high energies such as they will soon become available at the Brookhaven Relativistic Heavy Ion Collider. The PCM describes the collision dynamics within the early and dense phase of the reaction in terms of the relativistic, probabilistic transport of perturbative excitations (partons) of the QCD vacuum. I will present an overview of the current state of the numerical implementations of this model, as well as its predictions for nuclear collisions at RHIC and LHC.

## 1 Introduction

Klaus Kinder-Geiger (KKG), who perished in the crash of Swissair flight 111 near Halifax on September 2, 1998, was a brilliant theoretical physicist and one of my dearest friends. He was one of those few human beings who are truly irreplaceable, not because no one else could carry on Klaus' research, but because of the unique way he did physics and almost everything else in life. Klaus combined the abstract mind of the scientist who works on deep and esoteric questions of nature with the wild mind of the artist who is driven to create and perform in extraordinary ways. Klaus became famous as a physicist for the work he did on the parton cascade model of relativistic nuclear reactions. But he was equally famous as an extravagantly creative and perceptive person in the communities where he lived and made friends. I always thought that knowing Klaus was the closest I would ever get to meeting in person one of the French existentialists whose novels I had read as a student.

As many of you know, Klaus very much liked to draw sketches of himself and the world around him. The one entitled "The Wunderbar World of KKG", featured prominently in his home page on the World Wide Web (Figure 1). It captures nicely how Klaus viewed himself: always inspired to explore unfamiliar territory, uncover new ideas, and gather new experiences. Klaus believed that what really counted in life was the unusual, and he dared to live by his belief. When he was still a young student at the University of Frankfurt, before I knew him, Klaus had tried his hand at painting. His paintings are highly expressive and almost haunting; having seen them once one is not likely



Figure 1: The “Wunderbar World of KKG” sketch greeting visitors to Klaus’ World Wide Web site at BNL.

to ever forget them. One of his paintings, shown in Figure 2, is particularly remarkable: it anticipates Klaus’ greatest contribution to physics. Clearly, the subject of the painting must be a parton cascade, as Klaus imagined it in his artistic mind in 1986.<sup>a</sup>

The hustler, I am convinced, is depicting Klaus himself. (It is anyone’s guess whom the two other figures represent. Some similarities with well known members of the heavy ion physics community are unmistakable.) Lest anyone doubts this interpretation of the painting, let me point out that it is signed “Cucurullo ’86”. Vincent de Cucurullo, short: Vinnie, was Klaus’ artistic pseudonym by which he signed his paintings. The name of his celebrated parton cascade model code, VNI, is a rendering of the artist’s name under the cloak of a scientific acronym.

## 2 The Parton Cascade Model

The parton cascade model (PCM) was proposed by Klaus and me<sup>1</sup> in 1990-91 and developed much further during the following years by Klaus in a remarkable series of papers<sup>2,3,4,5</sup>. Our aim was to describe the energy deposition, thermalization, and chemical equilibration of matter in high energy nuclear collisions, and to provide a full space-time picture of the collision up to the moment when individual hadrons are formed. The model was, at least originally, not conceived as an “event generator” that would predict a full set of

<sup>a</sup>Of course, no one - not even Klaus himself - would have had any notion of this at the time the scene was painted. But the artist sometimes envisions novel ideas and concepts long before the scientist. What is unusual here is that the artist became the scientist!



Figure 2: The Pool Players. By *Vincent De Cucurullo*, 1986

hadron momentum distributions in the final state.

In order to enable experimental predictions, Klaus decided to develop the parton cascade model code VNI, which contains the implementation of a hadronization scheme in the framework of a parton coalescence model. This required a number of compromises and ran somewhat counter to the original purpose of the PCM, namely, to explore the range of validity of perturbative QCD in nuclear reactions. Nevertheless, the predictions of the PCM with an added hadronization stage have been, and continue to be, very useful. We simply do not know how to do better at the present time.

The conceptual basis of the PCM is the inside-outside cascade model<sup>6</sup> of high-energy hadron reactions, which implements the concept that new matter produced in hadronic interactions at high energy is formed outside the intersecting world-tubes of the colliding hadrons. Bjorken's hydrodynamical model<sup>7</sup> was devised to describe the evolution of this newly formed matter in the central space-time region after thermal equilibration. Beginning in the mid-1980's it was realized that the deposition of energy into this region may be, at least partially, described in terms of concepts based on perturbative QCD (minijets) when the energy of colliding heavy nuclei becomes very large.<sup>8,9,10,11</sup>

A computer code (HIJING) incorporating some of these ideas was developed by Gyulassy and Wang.<sup>12,13</sup>

The parton cascade model combined these ideas into one unified scheme for the description of the space-time evolution of matter in nuclear reactions. Its three main ingredients are:

1. The *initial state* is viewed as incoherent ensemble of partons determined by the nuclear parton distribution functions  $q_f(x, Q^2)$  and  $g(x, Q^2)$ , where the subscript  $f$  denotes the quark flavor and  $g(x, Q^2)$  stands for the gluon distribution.  $x = p_z/P$  is the longitudinal momentum fraction of the nucleon carried by the parton, and  $Q^2$  is the parton “scale” or virtuality. Before any interaction occurs,  $Q^2$  is generally taken as space-like. Our knowledge about the space-time structure of the nuclei before the collision and our limited information about the intrinsic transverse momenta of partons is then used to construct a model for the six-dimensional phase space distributions of partons before the interaction:  $q_f(r, p), g(r, p)$ . The parton distributions are conveniently initialized at the scale  $Q_0^2 = \langle p_T^2 \rangle_{\text{coll}}$  of the average momentum scale of the primary parton-parton interactions.
2. The *time evolution* of the parton phase distributions is governed by a relativistic Boltzmann equation with “leading-log” improved lowest-order collision terms. Only binary interactions are allowed, but the final state can have (and generally has) more than two particles. As is well known, the higher-order improvement of the cross sections by means of the leading logarithmic approximation is equivalent to the scale evolution of the parton distributions according to the DGLAP equation. Motivated by quantum mechanical considerations, the space-time picture of parton propagation before and after interactions is closely related to their off-shell propagation: the formation of a parton with virtuality scale  $Q$  takes a time  $\tau_f(Q) \approx \hbar/Q$ .
3. When the parton distributions become sufficiently dilute, they *hadronize*. In VNI the hadronization is described by a clustering algorithm, followed by the decay of excited hadrons. The transition is assumed to occur when the average virtuality of the partons falls below a critical value  $Q_{\text{crit}} \approx 1$  GeV, because partons no longer scatter with sufficient energy to allow for the collisions to be described by perturbative QCD.

From a gradient expansion of the evolution equation for the parton Wigner

distribution two equations can be derived.<sup>14</sup> The first equation

$$p^\mu \frac{\partial}{\partial r^\mu} F_i(r, p) = C_i(r, p) \quad (1)$$

describes the free propagation of partons which is intermittently modified by interactions given by the binary collision terms  $C$ . The second equation

$$p^2 \frac{\partial}{\partial p^2} F_i(r, p) = S_i(r, p) \quad (2)$$

describes the evolution of the parton distributions with respect to virtuality or “off-shellness”  $p^2$ . This equation is a generalization of the usual mass-shell condition  $F(r, p) \sim \delta(p^2 - m^2)$  to the case where the on-shell particle distribution cannot be defined.  $S_i(r, p)$  describes the splitting of single off-shell partons into two partons of smaller virtuality.

The two equations can be viewed as quantitative representation of Feynman diagrams of the type shown in Fig. 3. The collision term  $C_i$  is represented by the binary collision diagram contained in the box at the center of the complex Feynman diagram, whereas the splitting terms  $S_i$  are represented by the branchings of the initial- and final-state partons.<sup>b</sup> The Feynman diagram of Fig. 3 is finite only if the virtualities of all final-state partons are limited from below by some infrared cut-off  $\mu^2$ . In an isolated event  $\mu^2$  describes the *hadronization scale*, i.e. the virtuality scale below which partons can no longer be considered as approximately free, perturbation quanta. In a dense medium, where partons rescatter often,  $\mu^2$  is determined by the frequency of rescatterings (see Section 3.2).

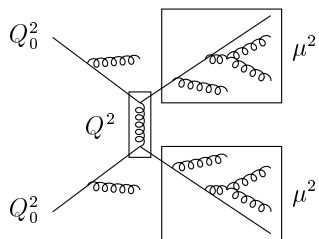


Figure 3: Graphical representation of the QCD transport equations (1,2) defining the parton cascade model. The  $2 \rightarrow 2$  scattering process at momentum scale  $Q^2$  is followed by the virtuality evolution from  $Q^2$  to  $\mu^2$ .

In the leading-logarithmic approximation (LLA), the differential cross section described by the Feynman diagram of Fig. 3 factorizes into a product of

<sup>b</sup>Strictly speaking,  $S_i$  describes the differential branching probability for an infinitesimal change in virtuality; the diagram is an integral representation of  $S_i$ .

terms for each of the two incoming and outgoing branch processes and one for the binary scattering process. Each branching term, in turn, is represented by a product of factors describing the individual branching events and the probabilities for the partons *not* to branch further in between. In other words, the Feynman diagram of the type shown in Fig. 3 defines a Markov process and, hence, can be described by a probabilistic one-body transport equation. This statement is no longer valid, if one tries to go beyond the LLA. However, certain effects beyond the LLA can still be described in terms of conditional probabilities, such as angular or  $k_T$ -ordering of gluons and certain soft-gluon interference effects. These effects are quantitatively important and have been incorporated into parton cascade codes.<sup>3</sup>

It is important to note that parton splittings are related by unitarity to loop diagrams that describe the running of the strong coupling constant  $\alpha_s(Q^2)$ . Both splittings and  $\alpha_s$ -running are described consistently in the LLA, which therefore satisfies the unitarity condition. In plain terms, the combined probability for all  $2 \rightarrow n$  parton diagrams with  $n \geq 3$  reduces the probability for the occurrence of a  $2 \rightarrow 2$  scattering and so on. By summing all  $2 \rightarrow n$  free diagrams, but not including the associated loop diagrams, unitarity would be violated. This violation leads to the divergence of the sum over  $n$  already at rather small parton center-of-mass energies, even in the presence of an infrared cut-off for the internal propagators.<sup>15</sup>

In the following sections I will discuss two important issues:

1. The space-time picture governing the initial-state parton distributions. This issue is closely connected with the problem of the decoherence of the initial parton wavefunctions.
2. The problem of infrared divergences of the perturbative parton cross sections. This issue is closely related to in-medium corrections of these cross sections, as well as to coherence properties of the initial-state wave functions.

The approach to local thermal equilibrium has been extensively studied within the framework of the parton cascade picture.<sup>13,16,17</sup> Without repeating the detailed arguments here, let me just state that the PCM approach predicts a very short kinetic equilibration time,  $\tau_{\text{th}} \ll 1 \text{ fm}/c$ , which is confirmed by full numerical calculations.<sup>2</sup>

### 3 Initial-state space-time picture

The probabilistic interpretation of parton distributions measured in deep-inelastic scattering is based on a summation over all final hadronic states.

A similar interpretation of one-body distributions arising in transport theory is grounded on the low-order truncation of the (BBGKY) hierarchy of Green functions and on an expansion in powers of  $\hbar$ . The validity of this picture ultimately relies on the separation of time scales in a dynamic process. Although these issues are generally well known,<sup>18</sup> their implications for nuclear parton cascades have not been fully explored. Recent advances<sup>19</sup> in our understanding of multiple scattering in QCD have shed some light on the intricacies of the formation time concept in non-abelian gauge theories, but it needs to be better understood how these results can be consistently incorporated into a probabilistic transport theory.

The original parton cascade model relied on some basic assumptions about initial parton distributions in space-time.<sup>16</sup> Denoting the parton light-cone momentum by  $p^+$ , the parton distributions were assumed to be distributed longitudinally according to the uncertainty relation:  $\Delta p^+ \Delta x^- \geq \hbar$ . Soft partons have the widest distributions in the variable  $x^-$ , and are assumed to travel both ahead of and behind the Lorentz contracted valence quark distributions. The argument is that this will not violate causality, because soft partons are emitted at a long distance before the collision and, travelling at the speed of light, can arrive significantly ahead of the quarks that emitted them.

The space-time picture of soft partons has been put on a much firmer foundation in recent years by the work of McLerran, Venugopalan and others on the random light-cone source model (RLSM).<sup>20</sup> In this model, one views the valence quarks constituting the fast-moving nucleus as a thin, Lorentz contracted sheet of locally random color sources. The color source is locally random, because valence quarks from several nucleons contribute at the same point in transverse space. The area density of color sources is given by  $\mu^2 = 3g^2 A / \pi R^2$ , where  $A$  is the nucleon number,  $R$  the nuclear radius, and  $g$  the QCD coupling constant. Clearly  $\mu$  grows as  $A^{1/6}$ , hence can be considered as a (potentially) large scale for sufficiently heavy nuclei.<sup>c</sup>  $\alpha_s(\mu^2) \ll 1$  can serve as the coupling parameter for a new type of perturbative expansion. Formally, the model maps into the problem of weakly coupled QCD in the presence of a random two-dimensional color source.

As shown by Kovchegov,<sup>21</sup> this model can be rigorously derived with standard light-cone techniques, which permit an explicit representation of the Gaussian ensemble of color sources. This representation can also be used to calculate the perturbative emission of soft gluons in collisions between two nuclei, described as encounter of two sheet-like clouds of valence quarks.<sup>22,23</sup> At

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<sup>c</sup> In practice,  $\mu \leq 1$  GeV even for the heaviest nuclei, even if the “hard” component of the gluon distribution is included in the color source.

leading order this soft gluon radiation is given by:

$$\frac{dN_g}{dyd^2k_\perp d^2b} = \frac{4\alpha_s^3}{\pi^2 k_\perp^2} \frac{N_c^2 - 1}{N_c} \langle T_{AB}(b) \rangle \int d^2g \frac{F(qa)F(|k-g|a)}{q^2(k-q)^2} \quad (3)$$

where  $F(qa)$  is the color-dipole form factor of the nucleon and  $T_{AB}(b)$  denotes the nuclear profile function. It can be shown<sup>24</sup> that this classical gluon radiation matches smoothly onto the perturbative minijet production of gluons at higher  $k_\perp$ .

Going beyond the classical approximation by including gluon-loop diagrams leads to a better and more rigorous understanding of the space-time distribution of soft gluons in a heavy nucleus.<sup>26</sup> The quantum corrections can be formulated in the framework of a space-time analogue of the renormalization group equations, describing the cascade of gluon emission leading to a power-law enhancement of soft gluons similar to the BFKL equation.<sup>27</sup> The RLSM approach also describes screening effects in the parton distribution at small  $k_\perp$ . The precise origin of this saturation has recently been elucidated by Kovchegov.<sup>28</sup>

The picture that emerges is the following: Gluons in the classical field generated by the valence quarks are fully Lorentz contracted by the Lorentz factor  $\gamma$  associated with the colliding nuclei, but gluons spawned by a splitting of those primary gluons experience only a partial Lorentz contraction of order  $x\gamma$ , where  $x$  is the momentum fraction carried by the parent gluon. As the branching process evolves to softer and softer gluons, the spatial extent of this gluon cloud becomes more and more diffuse in the light-cone variable  $x^-$ . This result confirms the intuitive picture embodied in the original parton cascade model, and provides a quantitative formulation of it.

#### 4 In-medium effects

In-medium effects on parton-parton interactions are essential to the viability of the parton cascade model. The application of perturbative QCD to nucleon-nucleon collisions requires the introduction of *ad hoc* cut-offs describing *nonperturbative* QCD effects, such as quark confinement and chiral symmetry breaking. In-medium effects, which grow rapidly in size as function of  $A$ , produce *perturbative* cut-offs when the density of the medium becomes sufficiently high. For example, QCD is known to become perturbative<sup>d</sup> at high temperature when the color-electric screening mass  $\mu_D \gg \Lambda_{\text{QCD}}$ .

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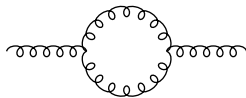
<sup>d</sup>Some nonperturbative effects remain even at high  $T$ , precisely because static magnetic interactions are not screened by perturbative in-medium interactions.



Although in-medium effects work in favor of the parton cascade model, they are not easily incorporated in practice. The problem is that in-medium effects are quite complicated and not easily treated correctly. The two main in-medium effects that are known to provide effective infrared cut-offs to perturbatively divergent parton interactions are:

- color-electric screening, which suppresses soft  $2 \rightarrow 2$  scattering amplitudes;
- gluon radiation suppression, which reduces  $2 \rightarrow 3$  (and  $2 \rightarrow n$ ) amplitudes with soft gluons in the final state.

Dynamical screening, at lowest order, is described by the in-medium contributions to the one-loop gluon polarization function:



At moderately high  $q_\perp$ , the gluon population grows like  $n(k) \approx (A_1 A_2)^{1/3}$ , providing a screening scale  $\mu(A)$  that increases rapidly with the size of the nuclei participating in the collision.<sup>31</sup>

Radiation suppression, also known as the Landau-Pomeranchuk-Migdal (LPM) effect, is a much more complicated mechanism. Its theoretical description requires a good understanding of the multiple scattering problem in QCD. Considerable progress has recently been made in this area, especially through the work of Baier et al.<sup>19</sup>, Zakharov<sup>32</sup> and others.<sup>33</sup> The main difference between QCD and the well-known case of QED is that a radiated gluon also rescatters in the medium at the same order in  $\alpha_s$  as the radiating particle; this is not so for a photon radiated by a fast charge moving in an electromagnetic plasma.

Revisiting the diagram shown in Fig. 3, the parton cascade model requires infrared cut-offs for both the central  $2 \rightarrow 2$  scattering matrix element and each of the four branching cascades. This is where the in-medium effects help: at high density one expects that the modifications of the elementary scattering amplitude ensure an infrared safe behavior. To date, two attempts have been made to practically implement the action of these in-medium effects:

1. In the self-screened parton cascade model (SSPCM<sup>34</sup>), the color-electric screening scale  $\mu(p_T)$  was calculated self-consistently for primary parton interactions only, and the further evolution of the parton plasma was described in the framework of the hydrodynamical model.

2. In Klaus' PCM code VNI<sup>35</sup>, the space–time picture of parton interactions is linked to the virtuality evolution of partons. A new interaction is permitted if its momentum transfer exceeds the virtuality scale of the participating partons at that time. Soft radiation is suppressed in the medium by this rule, because intermediate collisions continue to reset the parton virtuality to that of the latest collision.

Both of these approaches are based on specific assumptions about the relation between the space–time and virtuality evolution of off-shell components of the parton distributions. One way to study this issue rigorously is the Wigner function representation. First results<sup>36</sup> obtained by this method are interesting, but do not yet fully address the complications encountered in a QCD parton cascade. A more direct approach to the problem of the space–time picture of off-shell quantum fluctuation is based on a modification of the QCD evolution equations to include an infrared scale.<sup>14,37</sup> In free space this infrared scale is determined by properties of the final state (hadronization scale); in a medium it is determined by screening effects.

#### 4.1 Self-screened parton cascade

Here one considers the scattering of an initial state parton as completed after a time  $\tau(p_T)$  which depends on the momentum transfer in the reaction. The uncertainty relation suggests  $\tau(p_T) \sim \hbar/p_T$ . (We will drop the factor  $\hbar$  in the following.) The scattered partons are then assumed to screen the scattering processes that involve a smaller momentum transfer:

$$\mu_D^2(p_T) = \frac{3}{\pi^2} \alpha_s(p_T^2) \int_{p_T}^{\infty} d^3k |\nabla_k n(k)|. \quad (4)$$

The density of partons scattered at  $p_T$  is, in turn, influenced by the screening because the differential cross section depends on  $\mu_0$ :

$$\frac{d\hat{\sigma}}{dp_T^2} \sim \frac{\alpha_s(p_T)^2}{(p_T^2 + \mu_D^2(p_T))^2} |M(\hat{s}, \hat{t})|^2. \quad (5)$$

If  $\mu_D(p_T)$  becomes large enough at low  $p_T$ , so that  $d\hat{\sigma}/dp_T^2$  remains perturbatively small, the coupled set of equations can be integrated down to  $p_T = 0$ . Since the rapidity density of scattered partons grows as  $(A_1 A_2)^{1/3} \ln \sqrt{s}$ , this condition requires large  $A$  and high energy. The SSPCM concept has recently been investigated more formally by Makhlin and Surdutovich<sup>37</sup> within the framework of the closed–time–path formalism for real-time Green functions.

In this framework, the final state mass-shell condition for emitted partons regulates the infrared divergences of single-particle correlation functions, such as the parton densities. Properly carried through, this concept leads to a self-consistent equation for the gluon screening mass (plasmon mass) similar to the one used in the SSPCM.

Quantitatively, one finds that  $\mu_D$  approaches about 1 GeV at low  $p_T$  in Au + Au collisions at RHIC energy (100 GeV/u) and 1.5 GeV at LHC energy (2.75 TeV/u). The differential minijet cross section as function of  $p_T$  peaks at about the same value, clearly showing the improved infrared behavior of the self-screened parton cascade. The total deposited energy within one unit of rapidity and after a characteristic formation time of 0.25 fm/c is  $\epsilon_0 \approx 60$  GeV/fm<sup>3</sup> (RHIC) and  $\epsilon_0 \approx 430$  GeV/fm<sup>3</sup> (LHC). The conditions established by the SSPCM can, therefore, be taken as initial conditions for the thermal and chemical evolution of a quasi-equilibrated parton plasma.

The kinetic equations for the evolution of such a plasma were derived by Biró et al.<sup>38</sup> and by Xiong and Shuryak<sup>15</sup>. Extensive calculations<sup>40</sup>, including longitudinal and transverse expansion, have shown that the plasma cools down to the critical temperature of QCD ( $T_c \approx 150$  MeV) after 5 fm/c (RHIC) and 10 fm/c (LHC). The emission of electromagnetic probes by such an evolving QCD plasma has also been calculated.<sup>40</sup>

#### 4.2 Monte-Carlo space-time cascade

The statistical implementation of parton cascades in the Monte-Carlo simulation code VNI<sup>35</sup> achieves an improved infrared behavior through heuristic rules that suppress certain interactions on the basis of kinematic considerations. The first rule asserts that independent scattering events involving the same parton require a sufficient time separation so that the time between scatterings is larger than the duration of the individual events. With the duration of an interaction again defined as  $\tau(p_T) \sim p_T^{-1}$ , where  $p_T$  is the momentum exchange, this requires that the time between interactions  $\Delta\tau > \tau(p_T)$ . Another way of ensuring this condition is to endow a parton after a scattering by  $p_T$  with an initial virtuality  $Q_0 = p_T$ , which then gradually decreases with time as  $Q(\tau) = Q_0\tau(p_T)/\tau$ . A subsequent scattering with  $p'_T$  requires that  $p'_T > Q(\tau)$  at the moment of the interaction. A second similar rule suppresses soft parton splittings in the presence of multiple scattering. Again, this rule can be formulated in terms of a parton virtuality that decreases with time between scatterings and is reset by each new interaction (see Fig. 4).

Although VNI still contains “arbitrary” infrared cut-off parameters (determined by a comparison with nucleon-nucleon interactions), these are needed to

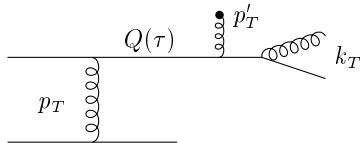


Figure 4: Illustration of in-medium suppression effects incorporated in the VNI code. The virtuality of a scattered parton evolves with time,  $Q(\tau)$ . Sequential scatterings or branchings are suppressed, if  $Q(\tau)$  is too large.

limit soft scatterings in the initial set of parton interactions, and they become important toward the end of the cascade evolution when the parton plasma becomes more and more dilute. The cut-off parameters effectively determine the end of the cascade evolution and, by suppressing soft interactions, they trigger the hadronization of parton clusters.

The fact that this improved parton cascade is much less dependent on arbitrary cut-off parameters makes it possible to explore applications of this model at lower energies where nucleon–nucleon collisions provide little information. Recently, Geiger and Srivastava<sup>41</sup> have studied the predictions that the VNI code makes for nuclear collisions in the energy regime of the CERN-SPS. While most of the particle yield at rapidities  $|y| \geq 2$  is produced by fragmentation of the unscattered beam remnants, the model predicts a significant contribution to particle production at central rapidity from partons that have undergone perturbative scattering.<sup>42</sup> This contribution is rising rapidly with nuclear mass  $A$ , roughly as  $(A_1 A_2)^{1/3}$ . This perturbative contribution to the energy deposition at  $|y| \leq 1$  coincides with a rapid increase of the energy density in scattered partons at  $\tau < 1$  fm/c, which rises from about 2 GeV/fm<sup>3</sup> in S+S to 5 GeV/fm<sup>3</sup> in Pb + Pb. This rise may be correlated with the much enhanced suppression of charmonium production in Pb + Pb collisions as observed by the NA50 experiment.<sup>43</sup>

## 5 Open problems

In spite of the considerable conceptual advances in the parton cascade model compared to its original formulation, certain aspects have still not been completely clarified. For issues concerning the initial state I can only refer to the recent review article of McLerran and Venugopalan.<sup>46</sup> One problem associated with the final state concerns the source of the produced entropy: The classical gluon field radiated in the encounter of two specified color charge distributions retains its full coherence during the course of the interaction.<sup>23</sup> The Gaussian average over initial conditions formally introduces entropy at late times, but a significant part of this entropy is already present in the initial state. The problem of thermalization remains a theoretical challenge even in this framework.

It is likely that its solution must be sought in the chaotic dynamical aspects of classical Yang-Mills fields that have been observed in numerical simulations of the classical non-abelian gauge theory.<sup>47,48</sup> The apparent decoherence of classical gauge fields is visible in a dramatic fashion when the collision of two non-abelian wave packets is studied in numerical simulations on the lattice.<sup>49</sup>

A consistent implementation of the RLSM into the parton cascade model requires a transport description of partons including mean color fields. Such a framework has been known for a long time<sup>50,51</sup>, but practical implementations of the QCD Boltzmann equation for partons and fields have been attempted only recently.<sup>52</sup> The idea behind this approach is to separate short-distance and long-distance dynamics by means of a lattice cut-off: excitations with momenta  $k \leq \pi/a$ , where  $a$  is the lattice spacing, are represented as classical fields on the lattice; those with higher momenta are represented as colored particles. The lattice cut-off must then be chosen such that  $g\mu < \pi/a < \mu$ . “Hard” collisions, i.e. those with a momentum transfer  $q > \pi/a$ , are now infrared safe due to the lattice cut-off, soft collisions are represented as interactions among particles via the lattice fields. These incorporate the crucial screening effects at the scale  $g\mu$ .

Another conceptual issue that is understood, in principle, but whose detailed investigation is an important outstanding problem, is the question what constitutes a consistent set of semiclassical transport equations for a theory with perturbatively massless modes such as QCD. This issue is conceptually resolved in the case of a massless scalar quantum field theory, where the introduction of a self-consistent medium-dependent mass term is sufficient. The issue is trickier in the case of QCD because of two problems: the need to retain gauge invariance and the long radiative tails of the spectral functions for the colored quasi-particles. Gauge invariance essentially requires that every modification of a  $n$ -body correlator is mirrored by an analogous modification of the  $(n + 1)$ -body correlator satisfying Ward identities. The treatment of radiative tails of the spectral function, on the other hand, requires renormalization group techniques.

At the time of his death, Klaus was working on this problem, applying the “exact” renormalization group technique<sup>53</sup> to QCD transport theory. On the evening before he boarded the ill-fated Swissair flight, he sent an almost finished draft of a manuscript<sup>54</sup> to Wetterich and me with the request: “I expect to see your comments when I return.” I believe that, again, Klaus was onto something very important, but it will remain for others to finish what he started. Let me just end this part of the discussion with the remark that the usual mass-shell condition in transport theory is intimately connected with the renormalization-group equations, and that a better understanding of this

connection is crucial to further progress in the quantum transport theory of QCD.

The next logical step in any parton cascade model that aims at practical predictions for experimentally observables is hadronization. In collaboration with John Ellis, Klaus worked intensely on this difficult problem.<sup>55</sup> Their work is based on an effective theory of scale invariance breaking in QCD and has had some success in describing hadronization of quark jets produced in  $e^+e^-$  annihilations. Further progress here will have to rely on an improved understanding of the mechanism of quark confinement in QCD.

Finally, let me point out that Steffen Bass had been in the midst of a collaboration with Klaus to add a state-of-the-art hadronic cascade model (UrQMD<sup>56</sup>) to the parton cascade code VNI, in order to describe final-state interactions of the hadrons created by hadronization of the quark-gluon plasma. The first results of this project are intriguing: they indicate that a small fraction of the created hadrons emerge without rescattering, whereas the bulk of the hadron yield is reprocessed through the melting pot of hadronic reactions.<sup>57</sup>

## 6 Summary

The parton cascade model was developed by Klaus to provide a QCD-based description of the approach to a locally thermalized state in collisions of heavy ions in the RHIC energy regime and beyond. In its original formulation the PCM predictions were critically dependent on several cut-off parameters that had to be determined from  $pp$  collision data. Recent advances incorporating in-medium effects into the parton interactions have reduced this dependence significantly, possibly allowing the application of the PCM over a wider energy range. Results obtained for nuclear collisions at CERN-SPS energies are intriguing.

The in-medium effects that modify parton-parton interactions not only reduce the parameter dependence of the model, they also provide valuable insight into the dynamics of a dense parton plasma. It is clear that we are just at the beginning here. The transport properties of off-shell quanta need to be understood much better, not only in cases where the off-shell propagator is dominated by a well-defined resonance, but especially in the case where the particles never get close to their mass shell as it applies to QCD. Another open question concerns the need for mean color fields. Such fields are not included in present versions of the PCM, but the random light-cone source model suggest that mean fields may be essential ingredients of a complete description of soft processes in nuclear collisions. One needs to take an average over a Gaussian ensemble of mean fields, where the width of the field distribution

is more important than the expectation value which remains zero. It would be interesting to explore possible connections of the RLSM to the traditional chromo-hydrodynamical model.<sup>44,45</sup>

Ultimately, the question is whether the parton cascade model can be replaced by a controlled approximation scheme where, in principle, successive orders of ever more sophisticated corrections can be calculated. We are still some steps away from a consistent formulation of transport phenomena off equilibrium in QCD. It is even unclear whether we even know what the small parameters in such an approximation scheme are. It is clear that a high density of excitations of the QCD vacuum is an essential condition, but there are many subtleties if one wants to go beyond this statement. However, steady progress in this field is being made, and there is reason to hope that a consistent formulation of transport phenomena in QCD can ultimately be achieved.

Finally, the treatment of the late phase of a relativistic heavy ion collision, when the dense matter breaks up into individual hadrons, has recently seen some exciting improvements. One still needs to rely on a phenomenological hadronization model for the transition to a hadronic cascade, but the latter can be studied with the technology that has been developed for the description of nuclear collisions at lower energies. In fact, the description of the hadronic final-state interactions should be much more reliable than, e.g. that of a nuclear collision at AGS energies, because the hadronic system starts out close to thermal equilibrium. Klaus was very excited about this approach, which will allow for a better comparison of the predictions of the parton cascade model with the experimental data that will soon become available from RHIC.

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